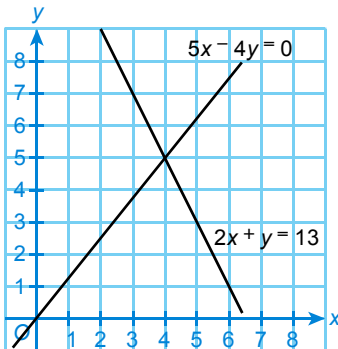




Fundamental Question

1. In each of the following, write down the solution of the simultaneous equations according to the figure given. (Give your answers correct to 1 decimal place if necessary.)

(a)
$$\begin{cases} 5x - 4y = 0 \\ 2x + y = 13 \end{cases}$$



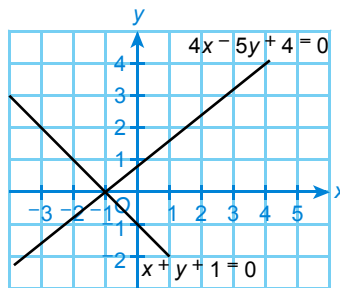
a) In the graph,
two straight lines intersect at
(4, 5).

\therefore The solution of the
simultaneous equations

$$\begin{cases} 5x - 4y = 0 \\ 2x + y = 13 \end{cases}$$

is $x = 4$, $y = 5$.

(b)
$$\begin{cases} x + y + 1 = 0 \\ 4x - 5y + 4 = 0 \end{cases}$$



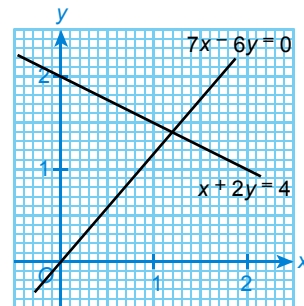
(b) In the graph,
two straight lines intersect at
(-1, 0).

\therefore The solution of the
simultaneous equations

$$\begin{cases} x + y + 1 = 0 \\ 4x - 5y + 4 = 0 \end{cases}$$

is $x = -1$, $y = 0$.

(c)
$$\begin{cases} 7x - 6y = 0 \\ x + 2y = 4 \end{cases}$$



(c) In the graph,
two straight lines intersect at
(1.2, 1.4) (corr. to 1 d.p.).

\therefore The solution of the
simultaneous equations

$$\begin{cases} 7x - 6y = 0 \\ x + 2y = 4 \end{cases}$$

is $x = 1.2$, $y = 1.4$.

2. Use the method of substitution to solve the following simultaneous equations.

(a)
$$\begin{cases} x = -5 \\ 4x + 5y = 0 \end{cases}$$

Substitute (1) into (2),

$$\begin{aligned} 4(-5) + 5y &= 0 \\ -20 + 5y &= 0 \\ 5y &= 20 \\ y &= 4 \end{aligned}$$

\therefore The solution of the simultaneous
equations is $x = -5$, $y = 4$.

(b)
$$\begin{cases} x = 5y - 10 \\ y - x = 6 \end{cases}$$

Substitute (1) into (2),

$$\begin{aligned} y - (5y - 10) &= 6 \\ y - 5y + 10 &= 6 \\ -4y &= -4 \\ y &= 1 \end{aligned}$$

Substitute $y = 1$ into (1),

$$\begin{aligned} x &= 5(1) - 10 \\ &= -5 \end{aligned}$$

\therefore The solution of the simultaneous
equations is $x = -5$, $y = 1$.

3. Use the method of elimination to solve the following simultaneous equations.

(a) $\begin{cases} x + y = -3 \\ x - y = -5 \end{cases}$

(b) $\begin{cases} x - y = 5 \\ x - 2y = -3 \end{cases}$

(c) $\begin{cases} -x + 3y = 6 \\ x + 2y = 4 \end{cases}$

a)(1) + (2),

$$(x + y) + (x - y) = -3 + (-5)$$

$$x + y + x - y = -8$$

$$2x = -8$$

$$x = -4$$

Substitute $x = -4$ into (1),

$$-4 + y = -3$$

$$y = 1$$

\therefore The solution of the simultaneous equations is $x = -4, y = 1$.

b)(1) - (2),

$$(x - y) - (x - 2y) = 5 - (-3)$$

$$x - y - x + 2y = 8$$

$$y = 8$$

Substitute $y = 8$ into (1),

$$x - 8 = 5$$

$$x = 13$$

\therefore The solution of the simultaneous equations is $x = 13, y = 8$.

c)(1) + (2),

$$(-x + 3y) + (x + 2y) = 6 + 4$$

$$-x + 3y + x + 2y = 10$$

$$5y = 10$$

$$y = 2$$

Substitute $y = 2$ into (2),

$$x + 2(2) = 4$$

$$x = 0$$

\therefore The solution of the simultaneous equations is $x = 0, y = 2$.

4. The difference between the ages of Jimmy and his mother is 22. Two years later, his mother's age will be 3 times Jimmy's. What is the current age of Jimmy?

Let x be the current age of Jimmy and y be the current age of his mother.

$$\begin{cases} y - x = 22 \dots\dots\dots (1) \\ y + 2 = 3(x + 2) \dots\dots\dots (2) \end{cases}$$

$$\begin{cases} y - x = 22 \dots\dots\dots (1) \\ y + 2 = 3(x + 2) \dots\dots\dots (2) \end{cases}$$

From (1), $y = 22 + x \dots\dots\dots (3)$

Substitute (3) into (2),

$$(22 + x) + 2 = 3(x + 2)$$

$$x + 24 = 3x + 6$$

$$2x = 18$$

$$x = 9$$

\therefore The current age of Jimmy is 9.



Consolidation Question

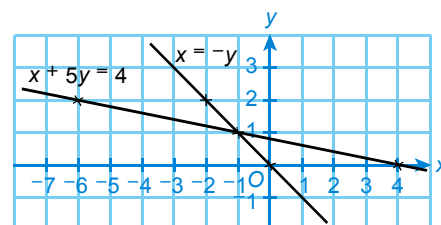
5. Solve the simultaneous equations $\begin{cases} x = -y \\ x + 5y = 4 \end{cases}$ graphically.

$$x = -y$$

x	-2	-1	0
y	2	1	0

$$x + 5y = 4$$

x	-6	-1	4
y	2	1	0



\therefore The solution of the simultaneous equations

is $x = -1, y = 1$.

6. Use the method of substitution to solve the following simultaneous equations.

(a) $\begin{cases} x + y = 1 \\ x - y = -3 \end{cases}$

- a) From (1), $x = 1 - y$ (3)

Substitute (3) into (2),

$$(1 - y) - y = -3$$

$$-2y = -4$$

$$y = 2$$

Substitute $y = 2$ into (3),

$$x = 1 - 2$$

$$= -1$$

\therefore The solution of the simultaneous equations is $x = -1, y = 2$.

(b) $\begin{cases} x + 2y = -1 \\ x - y = 5 \end{cases}$

- b) From (1), $x = -1 - 2y$ (3)

Substitute (3) into (2),

$$(-1 - 2y) - y = 5$$

$$-3y = 6$$

$$y = -2$$

Substitute $y = -2$ into (2),

$$x - (-2) = 5$$

$$x = 3$$

\therefore The solution of the simultaneous equations is $x = 3, y = -2$.

7. Use the method of substitution to solve the following simultaneous equations.

(a) $x - 3y - 6 = 2x - 9y - 9 = 0$

Rewrite the simultaneous equations as follows:

$$\begin{cases} x - 3y - 6 = 0 & \text{..... (1)} \\ 2x - 9y - 9 = 0 & \text{..... (2)} \end{cases}$$

From (1), $x = 3y + 6$ (3)

Substitute (3) into (2), $2(3y + 6) - 9y - 9 = 0$

$$6y + 12 - 9y - 9 = 0$$

$$-3y = -3$$

$$y = 1$$

Substitute $y = 1$ into (3),

$$x = 3(1) + 6$$

$$= 9$$

\therefore The solution of the simultaneous equations is $x = 9, y = 1$.

8. A stall sold 330 copies of Star Daily News and Mango Daily News altogether in a day with the total income of \$1 800. If the selling prices of Star Daily News and Mango Daily News are \$5 and \$6 each copy respectively, how many copies of each kind of newspapers did the stall sell during that day?

Let x be the number of copies of Star Daily News sold and y be the number of copies of Mango Daily News sold during that day.

$$\begin{cases} x + y = 330 & \text{..... (1)} \end{cases}$$

$$\begin{cases} 5x + 6y = 1\,800 & \text{..... (2)} \end{cases}$$

$$(1) \times 5, \quad 5x + 5y = 1\,650 \quad \text{..... (3)}$$

$$(2) - (3), \quad (5x + 6y) - (5x + 5y) = 1\,800 - 1\,650$$

$$5x + 6y - 5x - 5y = 150$$

$$y = 150$$

Substitute $y = 150$ into (1),

$$x + 150 = 330$$

$$x = 180$$

\therefore 180 copies of Star Daily News and 150 copies of Mango Daily News were sold during that day.

9. 40 employees of a company are evenly divided into 4 groups where the number of male employees in each group is 2 more than the number of female employees. How many male and female employees are there among the 40 employees?

Suppose there are x male employees and y female employees among the 40 employees,

$$\begin{cases} x + y = 40 & \text{..... (1)} \end{cases}$$

$$\begin{cases} \frac{x}{4} - \frac{y}{4} = 2 & \text{..... (2)} \end{cases}$$

$$(2) \times 4, \quad 4 \times \frac{x}{4} - 4 \times \frac{y}{4} = 4 \times 2$$

$$x - y = 8 \quad \text{..... (3)}$$

$$(1) + (3), \quad (x + y) + (x - y) = 40 + 8$$

$$x + y + x - y = 48$$

$$2x = 48$$

$$x = 24$$

Substitute $x = 24$ into (1),

$$24 + y = 40$$

$$y = 16$$

\therefore There are 24 male employees and 16 female employees among the 40 employees.

10. Use the method of elimination to solve the following simultaneous equations.

(a) $\begin{cases} 2x + y = 4 \\ x + 2y = 5 \end{cases}$

(1) $\times 2$, $4x + 2y = 8$ (3)
(3) $-$ (2),

$$(4x + 2y) - (x + 2y) = 8 - 5$$

$$4x + 2y - x - 2y = 3$$

$$3x = 3$$

$$x = 1$$

Substitute $x = 1$ into (1),

$$2(1) + y = 4$$

$$y = 2$$

\therefore The solution of the simultaneous equations is $x = 1$, $y = 2$.

(b) $\begin{cases} 4x + 9y - 42 = 0 \\ 7x + 3y + 3 = 0 \end{cases}$

(2) $\times 3$, $21x + 9y + 9 = 0$ (3)
(3) $-$ (1),

$$(21x + 9y + 9) - (4x + 9y - 42) = 0$$

$$21x + 9y + 9 - 4x - 9y + 42 = 0$$

$$17x + 51 = 0$$

$$17x = -51$$

$$x = -3$$

Substitute $x = -3$ into (2),

$$7(-3) + 3y + 3 = 0$$

$$3y = 18$$

$$y = 6$$

\therefore The solution of the simultaneous equations is $x = -3$, $y = 6$.

(c) $\begin{cases} 2y + 3x + 4 = 0 \\ 2x - 3y + 7 = 0 \end{cases}$

(1) $\times 3$, $9x + 6y = -12$ (3)
(2) $\times 2$, $4x - 6y = -14$ (4)
(3) $+$ (4),

$$13x = -26$$

$$x = -2$$

Substitute $x = -2$ into (1),

$$2y + 3(-2) + 4 = 0$$

$$2y = 2$$

$$y = 1$$

\therefore The solution of the simultaneous equations is $x = -2$, $y = 1$.



Challenging Question

11. Solve the simultaneous equations $\begin{cases} 2x + y = 3 \\ x + 2y = 8 \end{cases}$ graphically. (Give your answer correct to 1 decimal place.)

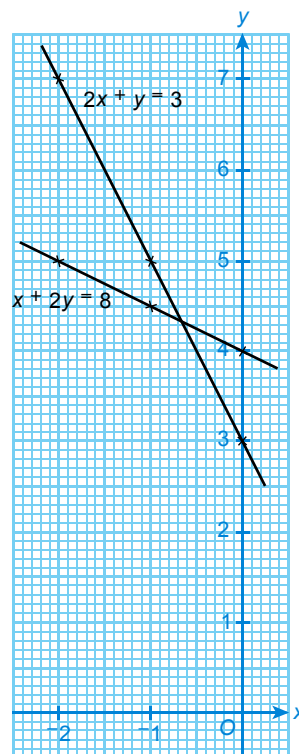
$$2x + y = 3$$

x	-2	-1	0
y	7	5	3

$$x + 2y = 8$$

x	-2	-1	0
y	5	4.5	4

\therefore The solution of the simultaneous equations is $x = -0.7$, $y = 4.3$ (corr. to 1 d.p.).



12. Use the method of substitution to solve the following simultaneous

equations.

$$(a) \begin{cases} 9x + 2y = 12 \\ 5(2x + 1) - y = -1 \end{cases}$$

a) From (2),

$$10x + 5 - y = -1 \\ y = 10x + 6 \dots\dots (3)$$

Substitute (3) into (1),

$$9x + 2(10x + 6) = 12$$

$$9x + 20x + 12 = 12$$

$$29x = 0$$

$$x = 0$$

Substitute $x = 0$ into (3),

$$y = 10(0) + 6$$

$$= 6$$

\therefore The solution of the simultaneous equations is $x = 0, y = 6$.

$$(b) \begin{cases} 2(x - y) - 5 = 0 \\ 5(x + y) + x = 4 \end{cases}$$

b) From (1),

$$x - y = \frac{5}{2}$$

$$y = x - \frac{5}{2} \dots\dots\dots (3)$$

Substitute (3) into (2),

$$5(x + x - \frac{5}{2}) + x = 4$$

$$5(2x - \frac{5}{2}) + x = 4$$

$$10x - \frac{25}{2} + x = 4$$

$$11x = \frac{33}{2}$$

$$x = \frac{3}{2}$$

Substitute $x = \frac{3}{2}$ into (3),

$$y = \frac{3}{2} - \frac{5}{2}$$

$$= -1$$

\therefore The solution of the simultaneous equations

is $x = \frac{3}{2}, y = -1$.

$$(c) \begin{cases} 5(x - y) + 2y = -26 \\ 2(x + 3y - 1) = y \end{cases}$$

c) From (2),

$$2x + 6y - 2 = y$$

$$2x = -5y + 2$$

$$x = \frac{-5y + 2}{2} \dots\dots (3)$$

Substitute (3) into (1),

$$5(\frac{-5y + 2}{2} - y) + 2y = -26$$

$$2[5(\frac{-5y + 2}{2} - y)] + 2(2y) = 2(-26)$$

$$5(-5y + 2) - 10y + 4y = -52$$

$$-25y + 10 - 10y + 4y = -52$$

$$-31y = -62$$

$$y = 2$$

Substitute $y = 2$ into (3),

$$x = \frac{-5(2) + 2}{2}$$

$$= \frac{-8}{2}$$

$$= -4$$

\therefore The solution of the simultaneous equations

is $x = -4, y = 2$.

13. Use the method of elimination to solve the following simultaneous equations.

$$(a) \begin{cases} \frac{4x+3y}{2} = 12 \\ 5x+3y = 39 \end{cases}$$

$$(1) \times 2, 2 \times \frac{4x+3y}{2} = 2 \times 12$$

$$4x + 3y = 24 \dots\dots\dots (3)$$

$$(2) - (3), (5x + 3y) - (4x + 3y) = 39 - 24$$

$$5x + 3y - 4x - 3y = 15$$

$$x = 15$$

Substitute $x = 15$ into (2),

$$5(15) + 3y = 39$$

$$3y = -36$$

$$y = -12$$

\therefore The solution of the simultaneous equations is $x = 15, y = -12$.

14. The number of pencils and ballpoint pens in a box is in the ratio of 3 : 2. If a pencil is put into the box and a ballpoint pen is taken out from it, the ratio will become 5 : 3. How many pencils and ballpoint pens are there in total in the box?

Let x be the number of pencils and y be the number of ballpoint pens in the box originally.

$$\left\{ \begin{array}{l} \frac{x}{y} = \frac{3}{2} \dots\dots\dots (1) \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{x+1}{y-1} = \frac{5}{3} \dots\dots\dots (2) \end{array} \right.$$

$$\text{From (1), } 2x = 3y \dots\dots\dots (3)$$

$$\text{From (2), } 3(x+1) = 5(y-1)$$

$$3x + 3 = 5y - 5$$

$$3x = 5y - 8 \dots\dots\dots (4)$$

$$(3) \times 3, 6x = 9y \dots\dots\dots (5)$$

$$(4) \times 2, 6x = 10y - 16 \dots\dots\dots (6)$$

$$(6) - (5), 6x - 6x = (10y - 16) - 9y$$

$$0 = 10y - 16 - 9y$$

$$y = 16$$

Substitute $y = 16$ into (3),

$$2x = 3(16)$$

$$x = 24$$

\therefore There are 24 pencils and 16 ballpoint pens in the box originally.

$$\begin{aligned} \therefore \text{ Total number of pencils and ballpoint pens } &= 24 + 16 \\ &= \underline{40} \end{aligned}$$